

Form factors for $B \rightarrow K l^+ l^-$ semileptonic decay from three-flavor lattice QCD

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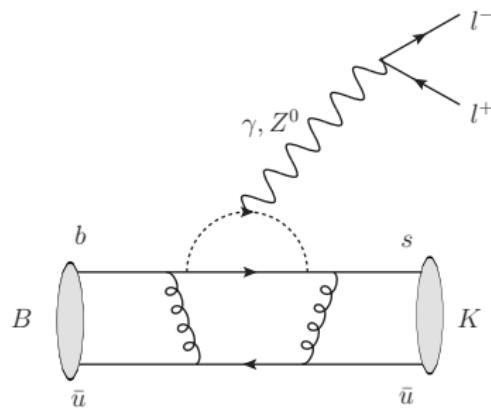
(In collaboration with FNAL/MILC)

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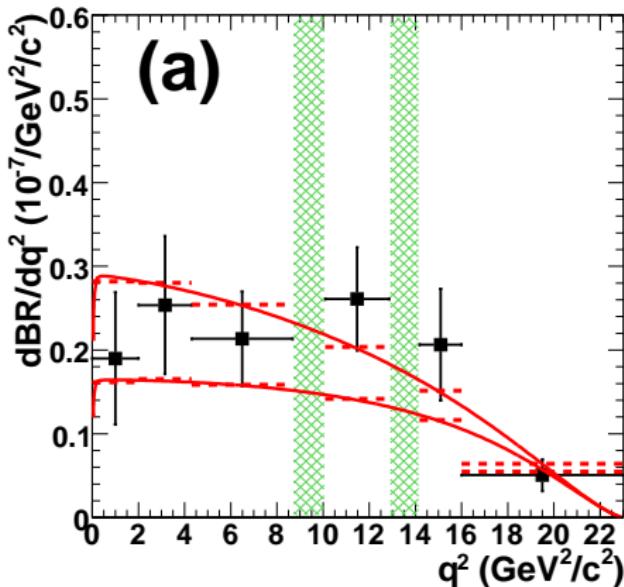
Motivations and theoretical background

$B \rightarrow K l l$ semileptonic decay occurs through Penguin diagram ($b \rightarrow s l l$).



- Standard Model (SM) contributes via FCNC (suppressed)
- Suitable process to detect physics BSM
- Studied by many experiment groups (BABAR, Belle, CDF, LHCb etc.)

Example of observable in $B \rightarrow K\ell\ell$ process



- $B^+ \rightarrow K^+ \mu^+ \mu^-$ differential branching ratio from CDF 2011
- Uncertainties in form factors are crucial to theoretical predictions (Red lines).
- High intensity front experiment (LHCb, SuperB) will come to with more accurate result.

Studies of $B \rightarrow K/\bar{K}$ form factors from lattice QCD

Quenched lattice QCD:

- A. Al-Haydari et al. (QCDSF) Eur. Phys. J. A 43, 107120 (2010)
- D. Becirevic et al. Nucl. Phys. B 769, 31 (2007)
- L. Del Debbio et al. Phys. Lett. B 416, 392 (1998)
- A. Abada et al. Phys. Lett. B 365, 275 (1996)

Recent studies on dynamical Nf=2+1 flavors ensembles:

- FNAL/MILC. ($B \rightarrow K/\bar{K}$) hep-lat/1111.0981
- Cambridge group. ($B \rightarrow K/K^*\bar{K}$) hep-ph/1101.2726

Lattice ensembles used in $B \rightarrow K\ell\ell$ work

$a^{-1}(fm)$	size	am_l/am_s	N_{meas}
0.12	$20^3 \times 64$	0.02/0.05	2052
0.12	$20^3 \times 64$	0.01/0.05	2259
0.12	$20^3 \times 64$	0.007/0.05	2110
0.12	$20^3 \times 64$	0.005/0.05	2099
0.09	$28^3 \times 96$	0.0124/0.031	1996
0.09	$28^3 \times 96$	0.0062/0.031	1931
0.09	$32^3 \times 96$	0.00465/0.031	984
0.09	$40^3 \times 96$	0.0031/0.031	1015
0.09	$64^3 \times 96$	0.00155/0.031	791
0.06	$48^3 \times 144$	0.0036/0.018	673
0.06	$64^3 \times 144$	0.0018/0.018	827

Table: Ensembles of QCD gauge field configurations used in the current B2K analysis. Four sources(0, $\frac{N_t}{4}$, $\frac{N_t}{2}$, $\frac{3N_t}{4}$) are used for all measurements

Form factors in $B \rightarrow K/\ell$ semileptonic decays

- Two matrix elements are needed in $B \rightarrow K/\ell$ work:

$$\langle B(p) | \bar{b} \gamma^\mu s | K(k) \rangle, \langle B(p) | \bar{s} \sigma^{\mu\nu} b | K(k) \rangle$$

$$\begin{aligned}\langle B(p) | \bar{b} \gamma^\mu s | K(k) \rangle &= f_+ (p^\mu + k^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu) + f_0 \frac{m_B^2 - m_K^2}{q^2} q^\mu \\ &= \sqrt{2m_B} \left[f_{\parallel} \frac{p^\mu}{m_B} + f_{\perp} p_{\perp}^\mu \right]\end{aligned}$$

$$\begin{cases} f_{\parallel}(E_K) = \frac{\langle B(p) | \bar{b} \gamma^0 s | K(k) \rangle}{\sqrt{2m_B}} \\ f_{\perp}(E_K) = \frac{\langle B(p) | \bar{b} \gamma^i s | K(k) \rangle}{2\sqrt{m_B}} \frac{1}{p_i} \end{cases}$$

$$\begin{cases} f_0(E_K) = \frac{2m_B}{m_B^2 - m_K^2} [(m_B - E_K) f_{\parallel}(E_K) + (E_K^2 - m_K^2) f_{\perp}(E_K)] \\ f_+(E_K) = \frac{1}{\sqrt{2m_B}} [f_{\parallel}(E_K) + (m_B - E_K) f_{\perp}(E_K)] \end{cases}$$

Form factors in $B \rightarrow K^{ll}$ semileptonic decays

Semileptonic $B \rightarrow K$ transition from tensor current:

$$q_\nu \langle K(k) | \bar{s} \sigma^{\mu\nu} b | B(p) \rangle = \frac{i f_T}{m_B + m_K} [q^2(p^\mu + k^\mu) - (m_B^2 - m_K^2) q^\mu]$$

Solve for f_T :

$$f_T = \frac{m_B + m_K}{\sqrt{2m_B}} \frac{\langle K(k) | i b \sigma^{0i} s | B(p) \rangle}{\sqrt{2m_B} k^i}$$

$B_x \rightarrow P_{xy} \parallel$ semileptonic decays in NLO SChPT

$$f_{\parallel} = \frac{C_0}{f} (1 + \text{logs} + C_1 m_x + C_2 m_y + C_3 E + C_4 E^2 + C_4 a^2)$$

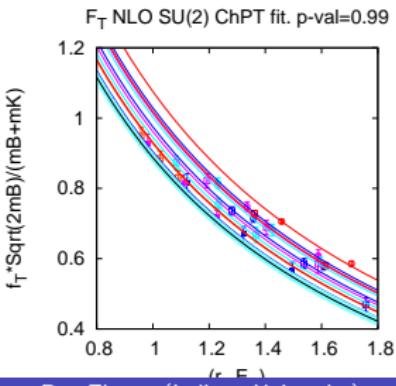
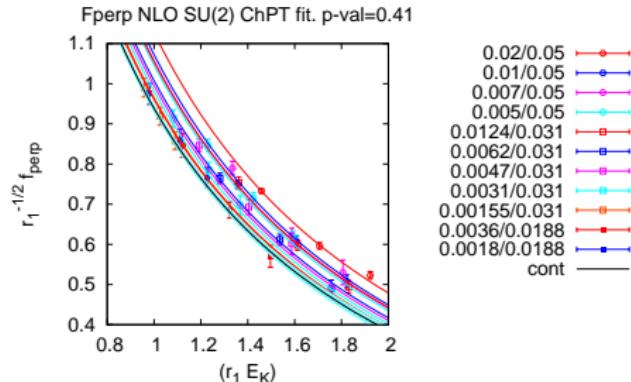
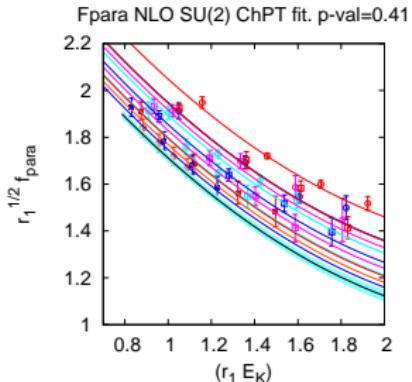
$$f_{\perp} = \frac{C_0}{f} \left[\frac{g}{E + \Delta_B^* + D} \right]$$

$$+ \frac{(C_0/f)g}{E + \Delta_B^*} (\text{logs} + C_1 m_x + C_2 m_y + C_3 E + C_4 E^2 + C_5 a^2)$$

where $\Delta_B^* = m_{B_s^*} - m_B$, D and logs are chiral log terms.

- We use SU(2) chiral logs in the chiral fit.
- The same formula are used for f_T and f_{\perp} .

f_{\parallel} , f_{\perp} and f_T chiral-continuum extrapolations



- Chiral-Continuum extrapolations give FFs at small E_K . (large q^2)
- $q^2 = (p_B - p_K)^2 = m_B^2 + m_K^2 - 2m_B E_K$
- z -expansion is a model independent extrapolation method to small q^2 .

z -expansion on $B \rightarrow Kll$ form factors

- z -expansion maps q^2 to z by:

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_- - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_- - t_0}}, \quad t_{\pm} = (m_B \pm m_K)^2$$

- Choose $t_0 = t_+ \left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right) \rightarrow z \in (-0.15, 0.15)$ for $B \rightarrow Kll$, corresponding $q^2 \in (0, 23)$.
- Expand form factors as a function of z .

$$f(q^2) = \frac{1}{B(z)\phi(z)} \sum_{k=0}^{\infty} a_k z^k,$$

where $B(z) = z(q^2, m_R^2)$ and $\phi(z)$ is selected such that $\sum_{k=0}^{\infty} a_k^2 \leq 1$

z -expansion on $B \rightarrow K/\ell$ form factors

$$f(q^2) = \frac{1}{B(z)\phi(z)} \sum_{k=0}^{\infty} a_k z^k,$$

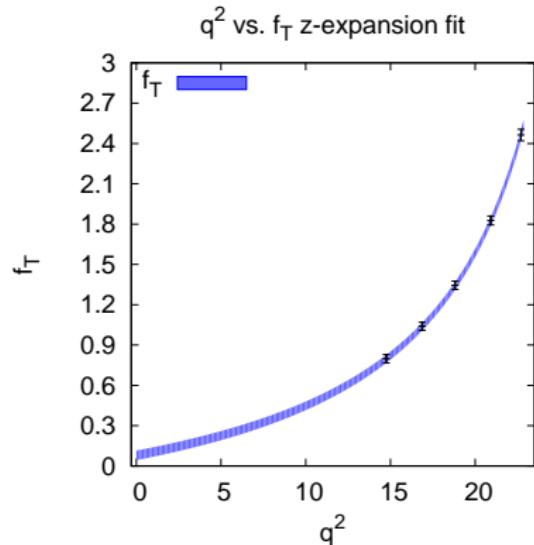
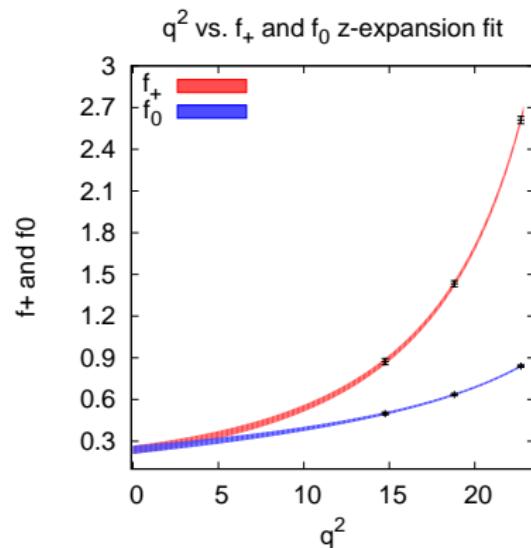
Numerical recipe for z -expansion:

- Fit $f(q^2)B(z)\phi(z)$ as a polynomial of z in the range of $z \in (-0.15, 0)$
- Extrapolate z -expansion fit to $z = 0.15$
- Convert variable z back to q^2

Now, we extend lattice measured form factors from $q^2 \in (15, 23) \text{ GeV}^2$ to whole q^2 range.

z -expansion on $B \rightarrow K/\ell$ form factors

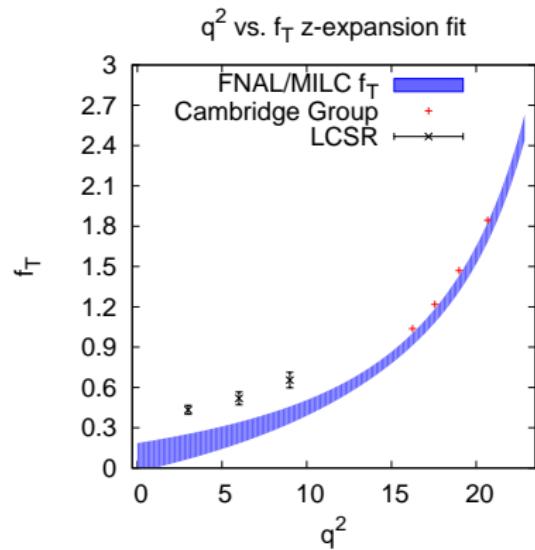
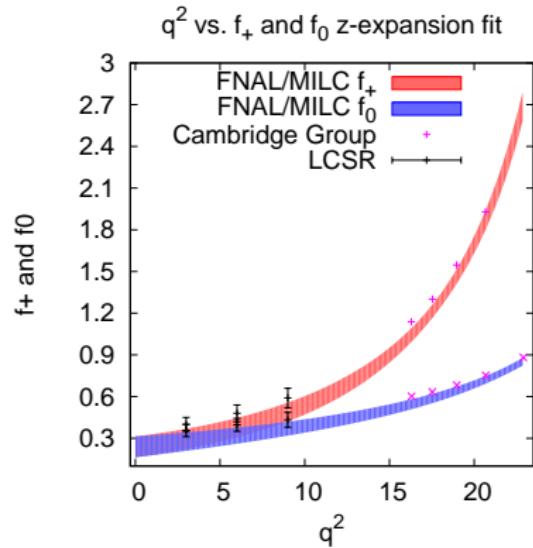
z -expansion on $B \rightarrow K/\ell$ form factors. (Statistical error only.)



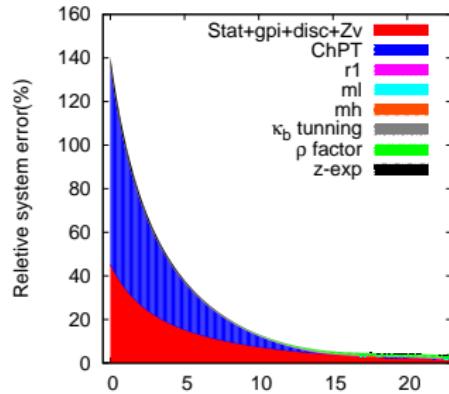
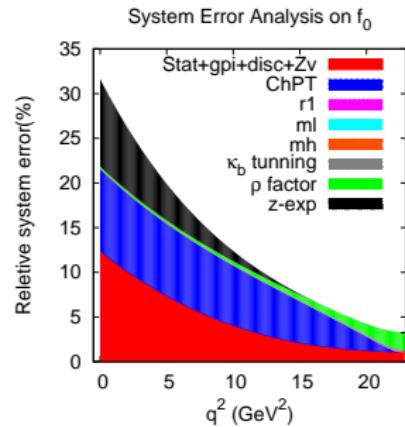
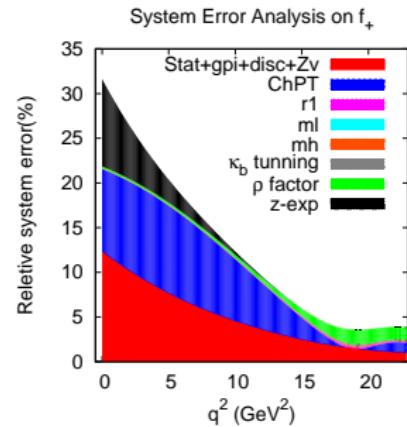
kinematic constraint, $f_+(q^2 = 0) = f_0(q^2 = 0)$, is applied in z -expansion fit.

z -expansion on $B \rightarrow K/\ell$ form factors

z -expansion on $B \rightarrow K/\ell$ form factors. (Statistical and systematic error.)



Systematic error budget



- Renormalization error could be smaller in the final result.
- ChPT error is important.
- Direct measurement of form factors at small q^2 is valuable.

Future work

- Study possible impact from the accurate form factors
- Consider to measure form factors at smaller quark masses and lower q^2 .
- Consider form factors in $B \rightarrow K^* ll$ semileptonic decay

Backup slides

f_{\parallel} SU(3) chiral log:

$$\begin{aligned} & \frac{1}{(4\pi f)^2} \left\{ \frac{1}{16} \sum_{\Xi} \left[\frac{2 - 3g_{\pi}^2}{2} I_1(m_{K,\Xi}) - 3g_{\pi}^2 I_1(m_{\pi,\Xi}) + \frac{1}{2} I_1(m_{S,\Xi}) \right. \right. \\ & + 2I_2(m_{K,\Xi}, E) + I_2(m_{S,\Xi}, E) \Big] \\ & - \frac{1}{2} I_1(m_{S,I}) + \frac{3g_{\pi}^2}{4} I_1(m_{\pi,I}) + \frac{8 - 3g_{\pi}^2}{12} I_1(m_{\eta,I}) + I_2(m_{\eta,I}, E) - I_2(m_{S,I}, E) \\ & + a^2 \delta'_V \left[\frac{I_1(m_{\eta',V}) - I_1(m_{\eta,V}) + I_2(m_{\eta',V}, E) - I_2(m_{\eta,V}, E)}{m_{\eta',V}^2 - m_{\eta,V}^2} \right. \\ & - \sum_{j \in \{S, \eta, \eta'\}} R_j^{[3,1]}(\{m_{S,V}, m_{\eta,V}, m_{\eta',V}\}; \{m_{\pi,V}\}) \left(\frac{1}{2} I_1(m_{j,V}) + I_2(m_{j,V}, E) \right) \\ & + \frac{3g_{\pi}^2}{2} \sum_{j \in \{\pi, \eta, \eta'\}} R_j^{[3,1]}(\{m_{\pi,V}, m_{\eta,V}, m_{\eta',V}\}; \{m_{S,V}\}) I_1(m_{j,V}) \Big] \\ & \left. + [V \rightarrow A] \right\} \end{aligned}$$

Backup slides

f_{\parallel} SU(2) chiral log:

$$\log s^{B \rightarrow K} = \frac{1}{(4\pi f)^2} \left\{ \frac{1}{16} \sum_{\Xi} \left[-3g_{\pi}^2 l_1(m_{\pi, \Xi}) \right] + \frac{3g_{\pi}^2}{4} l_1(m_{\pi, I}) \right. \\ \left. + \frac{3g_{\pi}^2}{2} [2l_1(m_{\pi, V}) - 2l_1(m_{\eta, V})] + [V \rightarrow A] \right\}$$